Big Idea Chapter 5 Review Guide

nth Roots and Rational Exponents

Key Concepts:

- $(\sqrt[n]{a})^m = a^{m/n}$ – this is the **key** relationship between n-th roots and rational exponents
- When n is **even** and a > 0, there are **two** real roots
  - Ex. $x^4 = 16$ has two real roots where $x = \pm 2$
- When n is **odd**, then there is **always** a single real root
  - Ex. $x^3 = 8$ has one real root where $x = 2$
  - Ex. $x^3 = -8$ has one real root where $x = -2$
- **All** exponential properties used for integer exponents apply for rational exponents
- $\sqrt[n]{x}$ is a single number!! $\sqrt[4]{16} = 2$ (not ±2)
- $\sqrt[n]{a * b} = \sqrt[n]{a} * \sqrt[n]{b}$ (assuming a>0 and b>0)
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- $\sqrt[a]{x^a} = |x|$ when a is an even number, $\sqrt[a]{x^a} = x$ when a is an odd number

Simplification rules:

- **Factor** the number inside the radical and then extract all the perfect powers according to the index
  - Ex. $\sqrt[4]{640000} = \sqrt[4]{2^6 * 10^4} = 10 * 2 * \sqrt[4]{4}$
- **Rationalize** the denominator by creating a perfect power inside the denominator's radical
  - Ex. $\frac{1}{\sqrt[5]{x^2}} = \frac{1}{\sqrt[5]{x^2}} * \frac{\sqrt[5]{x^3}}{\sqrt[5]{x^3}} = \frac{\sqrt[5]{x^3}}{\sqrt[5]{x^5}} = \frac{\sqrt[5]{x^3}}{x}$
  - Example of multiplying by conjugate: $\frac{1}{3+\sqrt{5}} * \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{3-\sqrt{5}}{3^2-\sqrt{5}^2} = \frac{3-\sqrt{5}}{4}$

Functions Operations / Composition

Domain of functions: the legal values of x

- Check that denominator is **not** 0
  - Ex. $f(x) = \frac{2x+3}{4x-5}$ so then $(4x - 5) \neq 0 \therefore \textit{the domain is all reals except } x \neq \frac{5}{4}$
- Check that numbers inside square root radicals are **non-negative** numbers
  - Ex. $f(x) = \sqrt{x + 3}$ so then $(x + 3) \geq 0 \therefore \textit{the domain is } x \geq -3$
- **Domain after function operation (e.g. (f+g)(x), (f-g)(x), (fg)(x) or (f/g)(x)**: Is the intersection of the domain of the original f(x) and g(x). Additionally, with (f/g)(x), x cannot be values where g(x)=0.

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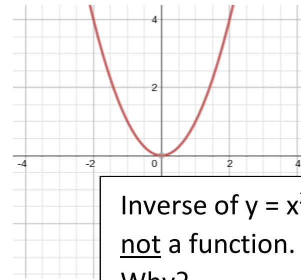
Inverse functions



Inverse of $y = x^2$ is <u>not</u> a function. Why?

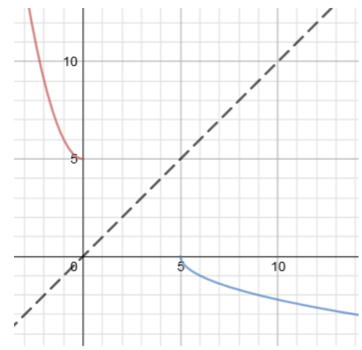
- The inverse function will "**undo**" / "**reverse**" the mapping of the function
  - Ex. If f(x) and g(x) are inverses of each other then:

    f(g(10)) = 10

    g(f(2.765)) = 2.765

    g(f(π)) = π
- **Definition & Test** for inverse functions f(x) and $f^{-1}(x)$
  - $f(f^{-1}(x)) = x$ <u>and</u> $f^{-1}(f(x)) = x$
- The inverse is the **reflection** of the function across the **y=x** axis
  - (x, y) → (y, x) is the mapping for a reflection <u>**and**</u> the method of generating the inverse

<u>Finding then Inverse</u>



- Swap the x and y's (for the equation and for the domain/range too‼)
- Solve for y

Example: $y = x^2 + 5$ domain: $x \le 0$ range: $y \ge 5$

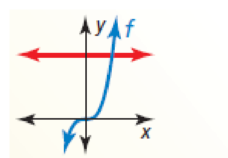
1) Swap x and y's: $x = y^2 + 5$

2) Solve for y: $y = \pm\sqrt{x-5}$ domain: $x \ge 5$ range: $y \le 0$

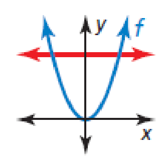
$y = -\sqrt{x-5}$ need to use only (-) version due to range constraint

<u>Testing if f(x)'s inverse is a function</u>

- If the horizontal line crosses **more than one point** in the graph, then the inverse will <u>**not**</u> be a function



Inverse is a function



Inverse is not a function

Solving radical / nth-root equations

- **Isolate** the radical term on one side of equation
- **Raise to the power of the reciprocal** ← **KEY STEP – this will turn the exponent into a 1**
- **Think REVERSE PEMA –** Parenthesis / Exponent / Multiplication / Addition
  - Ex.

$5(x+1)^{\frac{3}{2}} - 2 = 18$ Deal with the **A**dditive term – i.e. add 2

$5(x+1)^{\frac{3}{2}} = 20$ Deal with the **M**ultiplicative term – i.e. divide by 5

$(x+1)^{\frac{3}{2}} = 4$ Deal with the **E**xponential term – i.e. raise both to 2/3 power

$((x+1)^{\frac{3}{2}})^{\frac{2}{3}} = 4^{\frac{2}{3}}$ <u>Note</u>: The exponent becomes 1 when multiplied by reciprocal

$(x+1) = 4^{\frac{2}{3}}$ Deal with the **P**arenthesis term – i.e. subtract 1

$x = 4^{\frac{2}{3}} - 1$

- **NOTE**: **Check** for **extraneous** solutions when squaring both sides where there is a variable on each side
  - Ex. Need to check for extraneous solutions: $x + 1 = \sqrt{7x+15}$

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- **REMINDER**: $(a+b)^2 \neq a^2+b^2$ ← **NEVER** EVER DO THIS!
  $(x+4)^2 \neq x^2+16$ ← **NEVER** EVER DO THIS!!
  $(\sqrt{x-2}+4)^2 \neq (x-2)+16$ ← **NEVER** EVER DO THIS EITHER!!

  $(a+b)^2 = a^2+2ab+b^2$ ← DO THIS INSTEAD!!
  $(x+4)^2 = x^2++8x+16$ ← AND THIS!!!
  $(\sqrt{x-2}+4)^2 = (x-2)+8\sqrt{x-2}+16$ ← AND THIS!!!!

Solving Radical Inequalities

1. Solve as with equality
2. Constrain your solution with the domain of the original expression in radical

EX

$$3\sqrt{x-1} \le 12$$
$$\sqrt{x-1} \le 4$$
$$x-1 \le 16$$
$$x \le 17$$

But, (x-1) ≥ 0 in original equation, so solution is 1 ≤ x ≤ 17

Graphing radical functions like $y=\sqrt{x}$ or $y=\sqrt[3]{x}$

KEY CONCEPT *For Your Notebook*

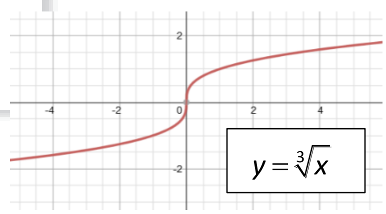
Graphs of Radical Functions

To graph $y = a\sqrt{x-h}+k$ or $y = a\sqrt[3]{x-h}+k$, follow these steps:

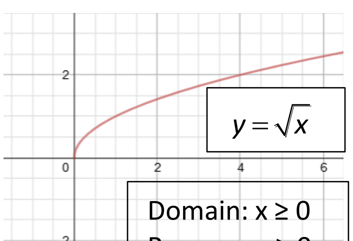
STEP 1 **Sketch** the graph of $y = a\sqrt{x}$ or $y = a\sqrt[3]{x}$.

STEP 2 **Translate** the graph horizontally h units and vertically k units.

KNOW THESE PARENT FUNCTION SHAPES!!!!

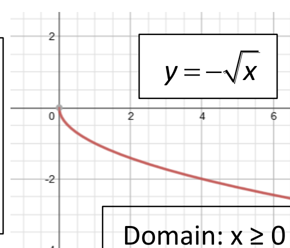


- Example: $y = 2\sqrt{x+6}-10$ will **translate** $y=2\sqrt{x}$ to **(-6, -10)**

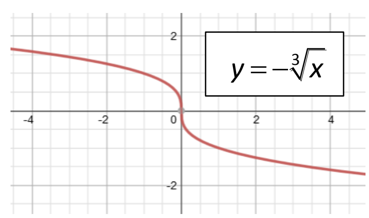


Domain: x ≥ 0
Range: y ≥ 0

Note: Top or bottom half of a side-ways parabola



Domain: x ≥ 0
Range: y ≤ 0



Domain: all R
Range: all R

Example transformation: $f(x)=\sqrt{x}$

1) Translate up by 4: $f_1(x) = f(x)+4 = \sqrt{x}+4$

2) Vertically stretch by 3: $f_2(x) = 3f(x) = 3\sqrt{x}+12$

4) Horizontally shrink by 1/5: $f_3(x) = f_2(5x) = 3\sqrt{5x}+12$

3) Translate left by 2: $f_4(x) = f_3(x+2) = 3\sqrt{5x+10}+12$